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A full-field perturbation approach to scattering and reverberation in range-dependent waveguides with rough interfaces

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Abstract

This paper describes a full-field perturbation approach to scattering and reverberation in complicated environments, such as range-dependent stratified media and waveguides with rough interfaces. Each interface is treated as a superposition of deterministic large-scale features (such as bathymetry changes) and random small-scale (comparable with the wavelength) roughness. Expressions for both reverberation field and average reverberation intensity in a general case of an arbitrary number of rough interfaces are obtained in a form, convenient for numerical simulations. In the case of long-range ocean reverberation, several approximations for these expressions are developed, relevant for various environmental scenarios and different types of interfaces, sea-surface, water-sediment interface, buried sediment interfaces, and bottom basement. The results are presented in a simple form and provide a direct relationship of reverberation intensity with three critical characteristics defined at each interface: (1) local spectra of small-scale roughness, (2) local contrast of acoustic parameters, and (3) two-way full-field transmission intensity calculated taking into account only large-scale changes of the environment.

Keywords: Reciprocity, integral equations, volume and roughness scattering, reverberation, range-dependent waveguides.

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I. INTRODUCTION

This paper develops a full-field model of reverberation in complicated environments, such as range-dependent stratified media and waveguides using a unified approach to volume and roughness scattering [1]. The approach was described in [1] in adequate detail only for the case of plane wave scattering from a heterogeneous fluid medium with rough interfaces plane-layered on the average. Such formulation usually appears in short-range bottom reverberation and direct path scenarios, where the description of acoustic bottom interaction is particularly built on conventional concept of scattering strength [2]. Using this concept assumes description of scattering strength as a function of grazing angles for incident and scattering waves, and therefore requires that incident waves be separated from the total field, which is not always possible, particularly in many cases relevant to long-range propagation and reverberation. A detailed discussion of this and other related issues can be found in [3], where it is suggested that other approaches, using full-field propagation and scattering models, are necessary to fully address the reverberation issues.

This paper develops further a full-field version of the unified approach [1] and extends it to the problem of long-reverberation in a waveguide with various rough interfaces, including sea surface, water-seafloor interface, internal (buried) sediment interfaces, and bottom basement. The problem formulation and equations developed in this article are general enough to consider range-dependent waveguides with complicated stratification and both large-scale (much larger than acoustic wavelength) interface deformations and small-scale (comparable with the wavelength) roughness. The large-scale features are treated deterministically, as spatial bathymetry changes, and are included in background or unperturbed environment, while the small-scale roughness is considered stochastically, as small random deformations superimposed on bathymetry and described by their local spectra. In this part, the model considered in this paper follows a two-scale approach used in many works on scattering from rough interfaces, see e.g. [4], [5], and discussion therein.

The results obtained in this paper are suggested to improve modeling and analysis of scattering in complicated environments, such as heterogeneous waveguides with rough interfaces, and particularly to better understand issues appearing in modeling of long-range reverberation. In particular, this can enhance existing models, such as UMPE/PEREV, the University of Miami PE (UMPE) model incorporating Tappert's PE Reverberation (PEREV) model, described in [3]. Results described in this paper are consistent with those given in [3], cast them as a particular case, and consider more general case of an arbitrary number of interfaces, including sea-surface and those buried in the sediment.

The paper is organized as follows. Section 2 gives formulation of the problem in terms of general (unified volume and roughness) perturbations, and provides general equations for the scattered field used in further analysis. In Sec.3, the problem is specified for the case of roughness perturbations and gives a general result. In Sec.4, a first-order solution is described, which allows simplification obtained general equations for various types of interfaces and comparing results with existing models. In Sec.5, a long-range approximation is described for scattering field, for various types of interfaces. Section 6 provides formulas for average reverberation intensity. Results are summarized in Section 7.

II. VOLUME PERTURBATIONS: RECIPROCITY THEOREM AND INTEGRAL EQUATION

A. Definitions and notations

Consider a heterogeneous fluid medium with spatially varying compressibility and density, $\kappa(\mathbf{r})$ and $\rho(\mathbf{r})$, respectively, with $\mathbf{r}$ being position vector. The spectral component of acoustic pressure field radiated by a point source located in position $\mathbf{A}$, at frequency ω, obeys equation

$$\nabla\cdot\left[\frac{1}{\rho(\mathbf{r})}\nabla p_A(\mathbf{r})\right]+\omega^2\kappa(\mathbf{r})\,p_A(\mathbf{r})=Q\delta(\mathbf{r},\mathbf{A}) \quad (1)$$

and the boundary conditions, continuity for pressure and for the normal component of the particle velocity at all interfaces. Factor Q is introduced here as a source strength factor, whose independence from position will be commented later on.

Pressure field is related with Green's function, which has unit field at unit distance, as follows

$$p_A(\mathbf{r})=D_A G(\mathbf{r},\mathbf{A}) \quad (2)$$

$$D_A=-\frac{Q\rho(\mathbf{A})}{4\pi} \quad (3)$$

so that for pressure field near the source we have

$$p_A(\mathbf{r})=-\frac{Q\rho(\mathbf{A})}{4\pi\,|\mathbf{r}-\mathbf{A}|},\ \mathbf{r}\to\mathbf{A} \quad (4)$$

Then, using the equation for particle velocity, $\rho\,\partial_t\mathbf{v}=-\nabla p$, one obtains

$$Q=Q=4\pi\,|\,\mathbf{r}-\mathbf{r}_A\,|^2\,\partial_t v_n=\partial_t\upsilon=-i\omega\upsilon \quad (5)$$

with υ being the source volume injection rate defined as the normal velocity on the spherical surface of small radius surrounding the source integrated over this surface.

The solution of (1), $p_A(\mathbf{r})$, will be called here unperturbed pressure, or the reference solution of the problem for the reference state of the medium, defined by its parameters $\kappa(\mathbf{r})$ and $\rho(\mathbf{r})$. This reference solution now will be compared with one, obtained for another state of the medium, called perturbed one, with parameters $\tilde{\kappa}(\mathbf{r})$ and $\tilde{\rho}(\mathbf{r})$, and for the same source, but located in different position $\mathbf{B}$. “Same” source here means “equivalent” one as defined in [6], i.e. with the same υ, and therefore same Q, which, along with following discussion of reciprocity, explains the choice of this parameter as a source position-independent strength factor.

B. Reciprocity and integral equations

Pressure field for perturbed state of the medium, or perturbed pressure, obeys equation

$$\nabla \cdot \left[\frac{1}{\tilde{\rho}(\mathbf{r})} \nabla \tilde{p}_B(\mathbf{r}) \right] + \omega^2 \tilde{\kappa}(\mathbf{r}) \tilde{p}_B(\mathbf{r}) = Q\delta(\mathbf{r}, \mathbf{B}) \tag{6}$$

Its relationship with unperturbed field can be obtained using following procedure [7-9]. Multiply (1) by $\tilde{p}_B$, subtract (6) multiplied by p_A, and integrate the result over a volume V, which contains both positions of the source, **A** and **B**, and all the interfaces. Take into account that

$$\tilde{p}_B \nabla \cdot \left(\frac{1}{\rho} \nabla p_A \right) - p_A \nabla \cdot \left(\frac{1}{\tilde{\rho}} \nabla \tilde{p}_B \right) = \\ \nabla \cdot \left(\tilde{p}_B \frac{1}{\rho} \nabla p_A - p_A \frac{1}{\tilde{\rho}} \nabla \tilde{p}_B \right) - \left(\frac{1}{\rho} - \frac{1}{\tilde{\rho}} \right) \nabla \tilde{p}_B \cdot \nabla p_A \tag{7}$$

For volume integration of the first term in right side of (7), use Gauss' theorem,

$\int_V \nabla \cdot \mathbf{F}\, dV = \oint_{S_V} \mathbf{F} \cdot \mathbf{dS}$, where the surface S_V includes both sides of each interface, so the sum of integrals over the two sides vanishes due to boundary conditions. Then only volume integral over the last term in (7) remains, resulting in an equation

$$\tilde{p}_B(\mathbf{A}) - p_A(\mathbf{B}) = \frac{1}{Q} \int_V \left[\omega^2 (\kappa - \tilde{\kappa}) p_A \tilde{p}_B - \left(\frac{1}{\rho} - \frac{1}{\tilde{\rho}} \right) \nabla p_A \cdot \nabla \tilde{p}_B \right] d^3 r \tag{8}$$

Equation (8) is exact and referred here as the reciprocity equation. It generates the two-fold consequence. First, conventional reciprocity is automatically manifested by equation (8) in the trivial case where the two states of the medium are acoustically equivalent, i.e. have equal acoustic parameters, which, with the integral in (8) vanished, results in

$$p_B(\mathbf{A}) = p_A(\mathbf{B}), \quad \tilde{p}_B(\mathbf{A}) = \tilde{p}_A(\mathbf{B}) \tag{9}$$

Another consequence appears as an integral equation for the field in perturbed medium, which follows from (8) with first of the reciprocity relations in (9) taken into account

$$\tilde{p}_B(\mathbf{A}) = p_B(\mathbf{A}) - \frac{1}{Q} \int_V \left[\omega^2 (\tilde{\kappa} - \kappa) p_A - \left(\frac{1}{\tilde{\rho}} - \frac{1}{\rho} \right) \nabla p_A \cdot \nabla \right] \tilde{p}_B \, d^3 r \tag{10}$$

The integral equation (10) is **exact** and provides a relationship between volume perturbations of the medium and the field change they cause.

If the unperturbed (or reference) medium is taken to be such that analytic or numerical solution for the reference field is known or can be easier obtained, e.g. a layered medium with homogeneous layers and smooth enough interfaces, then an approximate solution of (10) can be obtained by iterations in the form of multiple scattering series, see [1] and Appendix A therein. If volume perturbations of the medium are small, the field perturbation, $u(\mathbf{A},\mathbf{B}) = \tilde{p}_B(\mathbf{A}) - p_B(\mathbf{A})$, can be approximated as a single-scattered field

$$u(\mathbf{A},\mathbf{B}) = \tilde{p}_B(\mathbf{A}) - p_B(\mathbf{A}) \approx -\frac{1}{Q}\int_V \left[\omega^2 \left(\tilde{\kappa} - \kappa\right) p_A p_B - \left(\frac{1}{\tilde{\rho}} - \frac{1}{\rho}\right) \nabla p_A \cdot \nabla p_B \right] d^3 r \qquad (11)$$

which is linear with respect to medium perturbations, and called Born approximation.

III. ROUGH INTERFACES

A. Definitions and general result

Following a unified approach to volume and roughness scattering [1], roughness can be treated as a form of volume (spatial) perturbations caused by deformation of a reference (or unperturbed) interface. The reference interface is assumed to be smooth enough, so that analytic or numerical solution for the unperturbed field in the vicinity of this interface is known.

Relationship between the reference interface, S, and the deformed (perturbed) interface $\tilde{S}$, can be given by their vertical elevation functions as follows

$$\begin{aligned} S &: z = h(\mathbf{R}) \\ \tilde{S} &: z = \tilde{h}(\mathbf{R}) \end{aligned} \qquad (12)$$

The space between $\tilde{S}$ and S specifies volume V in (10), where volume perturbations appear to be of non-zero values. Within this volume, the differences of parameters, $\tilde{\kappa} - \kappa$ and $\tilde{\rho}^{-1} - \rho^{-1}$, have different sign above and below the reference interface, which however is compensated by changing sign of the integral over z due to general relation $\int_h^{\tilde{h}} [...]dz = -\int_{\tilde{h}}^{h} [...]dz$.

Let acoustic parameters be specified by indexes 1 and 2 for the medium above and below the interface, respectively. Then equation (10) becomes of the form

$$\tilde{p}_B(\mathbf{A}) = p_B(\mathbf{A}) - \frac{1}{Q}\int d^2R \int_{h(\mathbf{R})}^{\tilde{h}(\mathbf{R})} \left[\omega^2 (\kappa_2 - \kappa_1) p_A \tilde{p}_B - \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right) \nabla p_A \cdot \nabla \tilde{p}_B \right] dz \qquad (13)$$

Equation (13) is **exact** and generalizes the result given in [1], where it was developed for a horizontal flat reference interface and using slightly different notations.

B. Small roughness

To derive an integral equation for the case of small roughness, it is important to note several properties of the integrand in equation (13). First, it is independent of the angular orientation of the coordinate system. Secondly, the deformation of interface, which defines the volume of integration in (13), keeps unchanged the product of perturbed and unperturbed parameters within this volume, e.g. $\tilde{\rho}\rho = \rho_1\rho_2$. Then for the integrand within the volume we have

$$\begin{aligned} &\nabla p_A \cdot \nabla \tilde{p}_B = \\ &\nabla_R p_A \cdot \nabla_R \tilde{p}_B + \partial_z p_A \partial_z \tilde{p}_B = \nabla_t p_A \cdot \nabla_t \tilde{p}_B + \partial_n p_A \partial_n \tilde{p}_B = \\ &\nabla_t p_A \cdot \nabla_t \tilde{p}_B + \rho_1\rho_2 \left(\frac{1}{\rho} \partial_n p_A \right) \left(\frac{1}{\tilde{\rho}} \partial_n \tilde{p}_B \right) \end{aligned} \qquad (14)$$

where $\partial_n = \mathbf{n}\cdot\nabla$ and $\nabla_t = \nabla - \mathbf{n}\partial_n$ denote normal derivative and tangent gradient, with $\mathbf{n} = (-\nabla h,1)\big/\sqrt{1+(\nabla h)^2}$ being unit normal vector at the reference interface. Boundary conditions applied to (14) ensure the continuity of integrand in (13) at the interfaces. In the case of small roughness, using this continuity, one obtains

$$\tilde{p}_B(\mathbf{A}) = p_B(\mathbf{A}) - \frac{1}{Q}\int \zeta(\mathbf{R})\left[\omega^2(\kappa_2-\kappa_1)p_A\tilde{p}_B - \left(\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)\nabla p_A\cdot\nabla\tilde{p}_B\right]_{z=h(\mathbf{R})} d^2R \qquad (15)$$

where $\zeta(\mathbf{R}) = \tilde{h}(\mathbf{R}) - h(\mathbf{R})$ is the roughness perturbation function. Note that this function is defined in vertical direction, and all the integrand is defined in the coordinate system which does not change along the reference interface. If observation point approaches the reference interface, equation (15) becomes an integral equation for the field at this interface.

IV. FIRST-ORDER APPROXIMATION

A. General result

First-order solution can be obtained from (15) using (14) where not only perturbed field, but also perturbed normal component of particle velocity is replaced by their unperturbed values (see also, for more comments, Appendix B in [1]), i.e.

$$\left(p_A\tilde{p}_B\right)_S \approx T(\mathbf{R}) = \left(p_A p_B\right)_{z=h(\mathbf{R})} \qquad (16)$$

$$\left(\nabla p_A\cdot\nabla\tilde{p}_B\right)_S \approx J(\mathbf{R}) = \left[\nabla_t p_A\cdot\nabla_t p_B + \rho_1\rho_2\left(\frac{1}{\rho}\partial_n p_A\right)\left(\frac{1}{\rho}\partial_n p_B\right)\right]_{z=h(\mathbf{R})} \qquad (17)$$

This results in following equations

$$u(\mathbf{A},\mathbf{B}) = \int \zeta(\mathbf{R})\phi_{AB}(\mathbf{R})d^2R \qquad (18)$$

$$-Q\phi_{AB}(\mathbf{R}) = \omega^2(\kappa_2 - \kappa_1)T(\mathbf{R}) - \left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)J(\mathbf{R}) \tag{19}$$

Because all terms in (17) are continuous and can be taken at either side of the interface, we will be using different combinations of the two sides, so that the chosen combination would either allow simplifying the form of equation or help to show connection with known results obtained for different particular cases. For instance, taking normal derivatives of the fields p_A and p_B in (17) at different sides, one obtains

$$J = (\nabla_t p_A \cdot \nabla_t p_B)_{z=h\pm 0} + \partial_n p_A\big|_{z=h\pm 0}\partial_n p_B\big|_{z=h\mp 0} = \nabla p_A\big|_{z=h\pm 0} \cdot \nabla p_B\big|_{z=h\mp 0} \tag{20}$$

This expression will be used later to consider scattering from "weak" sediment interfaces. In some other cases, e.g. at the oceanic waveguide boundaries, sea-surface and bottom basement, using one-side boundary conditions may be preferable.

B. Ideal boundaries

Suppose S is a hard interface with Neumann boundary conditions $\partial_n p_{A,B}\big|_S = 0$. Then from (19) and (20) we have

$$Q\rho_1(S)\phi_{AB}(S) = \left[k_1^2 p_A p_B - \nabla_t p_A \cdot \nabla_t p_B\right]_{z=h+0} \tag{21}$$

where $k_1 = \omega(\kappa_1\rho_1)^{1/2}$ is the wave number in the medium above the interface, and it is assumed that $\kappa_2 << \kappa_1$, and $\rho_2 >> \rho_1$.

The expression for the field scattered from rough soft surface, e.g. sea-surface, can be obtained directly from (19) with Dirichlet boundary conditions $p_{A,B}\big|_S = 0$. In this case, assuming that $\rho_2 << \rho_1$, one obtains

$$Q\rho_1(S)\phi_{AB}(S) = \left[\partial_n p_A \partial_n p_B\right]_{z=h+0} \tag{22}$$

C. Arbitrary number of rough interfaces

Consider a general case of an arbitrary number of rough interfaces, i.e. $S = \sum_j S_j$ with functions $\zeta_j(S)$ defined at each interface S_j. Then, generalizing equation (18), we have

$$u(\mathbf{A},\mathbf{B}) = \sum_j u_j(\mathbf{A},\mathbf{B}) \tag{23}$$

$$u_j(\mathbf{A},\mathbf{B}) = \int \zeta_j(\mathbf{R}) \phi_{AB}^{(j)}(\mathbf{R}) d^2 R \tag{24}$$

$$-Q\rho_0 \phi_{AB}^{(j)} = k_0^2 \left(\frac{\kappa_{j+1} - \kappa_j}{\kappa_0} \right) T_j - \left(\frac{\rho_0}{\rho_{j+1}} - \frac{\rho_0}{\rho_j} \right) J_j \tag{25}$$

$$J_j = \left[\nabla_t p_A \cdot \nabla_t p_B + \rho_j \rho_{j+1} \left(\frac{1}{\rho} \partial_n p_A \right) \left(\frac{1}{\rho} \partial_n p_B \right) \right]_{S_j} \tag{26}$$

$$T_j(\mathbf{R}) = \left(p_A p_B \right)_{S_j} \tag{27}$$

where $k_0 = \omega / c_0$, c_0, κ_0, and ρ_0 are introduced as some space-independent values, which can be chosen as typical for the environment of interest, for wave number, sound speed, compressibility and density, respectively.

D. Plane waves in plane-layered reference medium

As an example, consider the case of plane waves in plane-layered medium with multiple flat interfaces $S_j : z = z_j$. In this case, solution for background field can be given in separable variables

$$p_{A,B}(\mathbf{r}) = \psi_{A,B}(z) \exp\left(i\mathbf{K}_{A,B} \cdot \mathbf{R} \right) \tag{28}$$

Then one obtains

$$-Q\phi_{AB}^{(j)}=\frac{k_0^2}{\rho_0}\left[\left(\frac{\kappa_{j+1}-\kappa_j}{\kappa_0}\right)+\left(\frac{\rho_0}{\rho_{j+1}}-\frac{\rho_0}{\rho_j}\right)\frac{\mathbf{K}_A\cdot\mathbf{K}_B}{k_0^2}+\left(\frac{\rho_{j+1}-\rho_j}{\rho_0}\right)Y_A^{(j)}Y_B^{(j)}\right]T_j \tag{29}$$

$$Y_{A,B}^{(j)}=\left[\frac{\rho_0\,\partial_z\psi_{A,B}}{\rho\,k_0\,\psi_{A,B}}\right]_{S_j} \tag{30}$$

It is easy to see that this result is consistent with obtained in [10] and described in more detail in [1] first-order perturbation solution for the case considered.

V. LONG-RANGE APPROXIMATION

A. General comments

In this section, we show that some of general formulas given in previous sections can be significantly simplified using assumptions relevant to long-range propagation scenarios. In particular, equations (25-27) can be cast as a more simple relationship

$$\phi_{AB}^{(j)}(S)=-2\frac{k_0^2}{Q\rho_0}\Gamma_j T_j \tag{31}$$

with Γ_j introduced as a slowly range-dependent factor, or bi-static “contrast parameter” defined at each interface. In following subsections, using relevant approximations for a number of different environmental scenarios, close forms for this factor are obtained and briefly discussed.

B. Preferable direction (small-angle approximation)

Consider a long-range propagation scenario, where approximations of PE-type for the propagation field can be applied as follows

$$p_{A,B}(\mathbf{r}) = \tilde{\psi}_{A,B}(\mathbf{R}, z)\exp\left(ik_0\mathbf{e}_{A,B} \cdot \mathbf{R}\right) \tag{32}$$

where k_0 is a reference wavenumber, $\mathbf{e}_{A,B}(\mathbf{R}) = \dfrac{\mathbf{R} - \mathbf{R}_{A,B}}{|\mathbf{R} - \mathbf{R}_{A,B}|}$ is a unit vector defining azimuthal direction from source (B) and receiver (A) to scattering point ($\mathbf{r}$), and it is assumed that wave numbers in the sediment are approximately equal to that in water near the sediment, and $k_0 \approx k_w$. Assume also that slopes of reference interfaces are small. In this case, directions of all waves propagated in the sediment are nearly horizontal, so that normal derivatives in (26) can be neglected, while for tangent derivatives following approximation can be applied

$$\nabla_t p_{A,B} \approx ik_0\mathbf{e}_{A,B} p_{A,B} \tag{33}$$

Then equation (25) becomes of the form (31) with

$$2\Gamma_j = \left(\frac{\kappa_{j+1} - \kappa_j}{\kappa_0}\right) + \left(\frac{\rho_0}{\rho_{j+1}} - \frac{\rho_0}{\rho_j}\right)\mathbf{e}_A \cdot \mathbf{e}_B \tag{34}$$

A similar “contrast parameter” was introduced by F.Tappert for the case of first (water-sediment) interface, see [3], with $\Gamma_b = \Gamma_1$. Equation (34) therefore consistent with Tappert’s result, generalizes it for the case of multiple interfaces, and is referenced here as Tappert’s approximation.

C. “Weak” sediment interfaces

Here we consider a possibility to take into account vertical derivatives in (26) using an approximation somewhat similar to (32), assuming that waves above and below the interface propagate in close directions but not necessarily horizontally. Instead of (32), we have now

$$\nabla p_{A,B}(\mathbf{r})_{z=h_j+0} \approx ik_j \mathbf{e}^{+}_{A,B}(\mathbf{r})\, p_{A,B}(\mathbf{r})_{z=h_j} \tag{35}$$

$$\nabla p_{A,B}(\mathbf{r})_{z=h_j-0} \approx ik_{j+1} \mathbf{e}^{-}_{A,B}(\mathbf{r})\, p_{A,B}(\mathbf{r})_{z=h_j} \tag{36}$$

where $\mathbf{e}^{+-}_{A,B}$ are unit vectors defined by directions of waves above and below interface from source and receiver to scattering point. Using boundary conditions in (20), we now obtain

$$\begin{aligned} 2\Gamma_j &= \left(\frac{n_j^2}{m_j} - \frac{n_{j+1}^2}{m_{j+1}}\right) + \left(\frac{1}{m_j} - \frac{1}{m_{j+1}}\right) n_j n_{j+1} \cos\vartheta_j = \\ &\left(n_j + n_{j+1}\right)\left(\frac{n_j}{m_j} - \frac{n_{j+1}}{m_{j+1}}\right) - 2 n_j n_{j+1} \left(\frac{1}{m_j} - \frac{1}{m_{j+1}}\right) \sin^2\left(\vartheta_j / 2\right) \end{aligned} \tag{37}$$

where $m_{j,j+1}$ and $n_{j,j+1}$ are introduced as density sediment/water ratio and refraction index in the sediment above and below the *j*-th interface, and $\cos\vartheta_j = \mathbf{e}^{+}_{A} \cdot \mathbf{e}^{-}_{B} = \mathbf{e}^{+}_{B} \cdot \mathbf{e}^{-}_{A}$. For the case of backscattering, where $\vartheta_j = 0$, equation (37) is given recently in [11]. It can be easily seen that equation (37) gives (34) in particular case of first (water/sediment) interface, if same other assumptions used in derivation of (34) are applied.

D. Basement interfaces

At the basement, a similar to (32) relation can be applied for horizontal derivatives, while vertical derivatives can be estimated assuming that only outgoing waves exist below the basement. In this case, taking into account boundary conditions, we have following approximations

$$\nabla_R\, p_{A,B}(\mathbf{r})_{z=h_N+0} \approx ik_N \mathbf{e}_{A,B}(\mathbf{R})\, p_{A,B}(\mathbf{r})_{z=h_N+0} \tag{38}$$

$$\begin{aligned}&\partial_z p_{A,B}(\mathbf{r})_{z=h_N+0} = \frac{m_{N+1}}{m_N}\partial_z p_{A,B}(\mathbf{r})_{z=h_N-0} \approx \\ &i\frac{m_{N+1}}{m_N}\sqrt{k_{N+1}^2 - k_N^2}\, p_{A,B}(\mathbf{r})_{z=h_N+0}\end{aligned} \tag{39}$$

resulting in the expression for rough sediment basement as follows

$$\begin{aligned}2\Gamma_N &= \left(\frac{n_N^2}{m_N} - \frac{n_{N+1}^2}{m_{N+1}}\right) + n_N^2\left(\frac{1}{m_N} - \frac{1}{m_{N+1}}\right)\cos\vartheta_N + \frac{m_{N+1} - m_N}{m_{N+1}^2}\left(n_{N+1}^2 - n_N^2\right) = \\ &\frac{m_N}{m_{N+1}^2}\left(n_N^2 - n_{N+1}^2\right) + 2n_N^2\left(\frac{1}{m_N} - \frac{1}{m_{N+1}}\right)\cos^2\left(\vartheta_N/2\right)\end{aligned} \tag{40}$$

In particular case with $N = 1$, where there is only water-basement interface, equation (40) results in

$$2\Gamma_1 = \frac{1}{m_b^2}\left(1 - n_b^2\right) + 2\left(1 - \frac{1}{m_b}\right)\cos^2\left(\vartheta/2\right) \tag{41}$$

where $m_b = m_2/m_1$ and $n_b = n_2/n_1$. It is interesting to compare this expression with Tappert's approximation. Noting that (34) can be presented in the form

$$2\Gamma_b = \frac{1}{m_b}\left(1 - n_b^2\right) + 2\left(1 - \frac{1}{m_b}\right)\cos^2\left(\vartheta/2\right) \tag{42}$$

it can be seen that (41) may provide a noticeable correction to Tappert's approximation given here by (42), if refraction indexes and densities of the bottom and water are different.

E. How to include rough sea-surface

It is easy to see that (22) is consistent with the expressions for the basement (41) and boundary conditions (39), that is sea-surface can be considered as a particular case of the basement with a very small basement/water density ratio.

Another way to include rough sea-surface results from approximating the vertical derivatives in (22) by a simple estimate $\partial_z p_{A,B}(\mathbf{r})_{z=z_S} \approx p_{A,B}(\mathbf{r})_{z=z_0=z_S+d} / d$, which provides following result for “effective contrast parameter” at a slightly shifted sea-surface defined as follows

$$S_0 : z_0 = z_S + d, \quad \Gamma_0 = (k_w d)^{-2} / 2 \tag{43}$$

Therefore, the scattered field in the oceanic waveguide with multiple rough interfaces can be presented uniformly for all interfaces, including sea surface, as a sum of fields scattered from different interfaces, $S_j,\ j = 0,\ldots,N$, using equations (23,24) and (31) with contrast parameters Γ_j defined by equations (37), (40) or (41), and (43).

VI. LONG-RANGE REVERBERATION INTENSITY

From results given in previous sections for the scattered field, the average scattering intensity can be calculated as follows. Using equation (23) results in

$$I = \sum_{j=1}^{N}\sum_{n=1}^{N}\left\langle \left| u_j(\mathbf{A},\mathbf{B}) u^*_n(\mathbf{A},\mathbf{B}) \right|^2 \right\rangle \tag{44}$$

where <…> denotes average over ensemble of small-scale roughness. Using (24), (31), and (3), one obtains

$$I = |D_0|^2\, k_0^4 \sum_{j,n} \iint g_{jn}(\mathbf{R},\mathbf{q}) \Gamma_j(\mathbf{R}) \Gamma^*_n(\mathbf{R}) \Phi_{jn}(\mathbf{R},\mathbf{q})\, d^2R\, d^2q \tag{45}$$

where $D_0 = -4\pi/(Q\rho_0) = D_A \rho(\mathbf{B}) / \rho_0 = D_B \rho(\mathbf{A}) / \rho_0$, Φ_{jn} is the local roughness cross-spectrum of *j*-th and *n*-th interfaces

$$\Phi_{jn}(\mathbf{R},\mathbf{q}) = (2\pi)^{-2} \int \left\langle \zeta_j(\mathbf{R} + \mathbf{a}/2) \zeta_n(\mathbf{R} - \mathbf{a}/2) \right\rangle \exp(i\mathbf{q}\cdot\mathbf{a})\, d^2a \tag{46}$$

and g_{jn} is a spatial Wigner-Ville distribution of normalized two-way propagation functions for these two interfaces

$$g_{jn}(\mathbf{R},\mathbf{q}) = (2\pi)^{-2}\int P_j(\mathbf{R}+\mathbf{a}/2)P^*_n(\mathbf{R}-\mathbf{a}/2)\exp(i\mathbf{q}\cdot\mathbf{a})\,d^2a \qquad (47)$$

$$P_j(\mathbf{R}) = T_j(\mathbf{R})\big/(D_A D_B) = \left[G(\mathbf{r},\mathbf{A})G(\mathbf{r},\mathbf{B})\right]_{S_j} \qquad (48)$$

Here we consider the case where there is no cross-correlation between roughness of different interfaces, which gives $\Phi_{jn} = \delta_{jn}\Phi_j$, with δ_{jn} being Kronecker delta. We assume also for simplicity that small-angle approximation (32) for propagation field is applicable. In this case, $g_{jj}(\mathbf{R},\mathbf{q})$ has a sharp local maximum, which can be approximated as follows

$$g_{jj}(\mathbf{R},\mathbf{q}) \approx \delta(\mathbf{q}-\mathbf{q}_S)\left|P_j(\mathbf{R})\right|^2 \qquad (48)$$

with $\mathbf{q}_S(\mathbf{R}) \approx k_0\mathbf{e}_A(\mathbf{R}) + k_0\mathbf{e}_B(\mathbf{R})$ being local scattering Bragg vector. Then, taking into account (2), we have $T_j(\mathbf{R}) = D_A D_B\left[G(\mathbf{r},\mathbf{A})G(\mathbf{r},\mathbf{B})\right]_{S_j}$, which results in an expression

$$I = |D_0|^2\sum_{j=1}^{N}\int\left|P_j(\mathbf{R})\right|^2 M_j(\mathbf{R})d^2R \qquad (49)$$

where M_j is introduced as the local scattering coefficient of *j*-th interface

$$M_j(\mathbf{R}) = \frac{1}{4}k_w^4\left|\Gamma_j(\mathbf{R})\right|^2\Phi_j(\mathbf{R},\mathbf{q}_S) \qquad (50)$$

defined by a local spectrum of roughness and local contrast of acoustic parameters which may vary along this interface, and factor $\left|P_j\right|^2$ describes the full-field intensity ensonifying this interface which may also vary significantly because of complicated effects of two-way propagation in the waveguide from source to scattering area and then to receiver, including effects of focusing and defocusing due to diffraction on large-scale variations of interfaces, 3D-refraction in water column and in the sediment, and others.

VII. SUMMARY AND CONCLUSIONS

This paper develops from first principles a unified perturbation approach to scattering in fluid media with volume heterogeneity and rough interfaces. A general reciprocity equation is used to derive an exact integral equation for the perturbation of pressure field caused by spatial perturbations of acoustic parameters of the medium, compressibility and density. An approximate solution can be obtained by iterations of the integral equation, with first iteration being the sum of the zeroth-order pressure field (unperturbed or reference one) and the first-order field (or Born approximation for scattered pressure).

Roughness of interfaces is treated as a form of volume perturbations caused by small-scale (comparable with the wavelength) deformation of a reference interface assumed to be much smoother to provide the reference field for the environment of interest. The first-order solution for pressure field is obtained for the general case with an arbitrary number of rough interfaces, various environmental scenarios and different types of interfaces, sea-surface, water-sediment interface, buried sediment interfaces, and bottom basement. Ideal interfaces, with Dirichlet and Neumann boundary conditions respectively, are shown to be particular cases of general results as well.

Average reverberation intensity is considered and a general result for an arbitrary number of rough interfaces is expressed through a matrix of roughness cross-spectra. For the case where there is no cross-correlation between roughness of different interfaces, the paper provides the direct relationship of reverberation with three critical characteristics defined at each interface: (1) local spectra of roughness, (2) local contrast of acoustic parameters, and (3) two-way transmission intensity of the reference field. The main physical implication is clear and general: the strongest returns will come from areas that are strongly ensonified, have greater interface contrasts of acoustic parameters, and larger roughness spectrum component at corresponding Bragg wave numbers, which confirms main conclusions made in previous work on full-field analysis of long-range reverberation, such as described in [3], and generalizes them to other environmental scenarios.

Note that the approach developed in this paper does not require Monte-Carlo simulations of the scattered field be used for analysis of average reverberation intensity. Also, the approach does not involve the concept of conventional scattering strength which requires extracting incident and scattered waves from full propagation fields and specifying their grazing angles at each interface, which is not always possible. This complication appears e.g. in the case, where grazing angles and interface slopes are both small and comparable to each other, i.e. in usual long-range propagation scenarios, where therefore full-field modeling approaches are required, such as the one developed in this paper. Besides, the approach provides the expressions for long-range reverberation in a simple form which includes only propagation intensity, i.e., neither the phases nor spatial derivatives of propagation field are needed to be accounted in the propagation model. These may provide a base for developing new algorithms for extremely fast calculations of reverberation in complicated range-dependent environments.

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14. ABSTRACT
This paper describes a full-field perturbation approach to scattering and reverberation in complicated environments, such as range-dependent stratified media and waveguides with rough interfaces. Each interface is treated as a superposition of deterministic large-scale features (such as bathymetry changes) and random small-scale (comparable with the wavelength) roughness. Expressions for both reverberation field and average reverberation intensity in a general case of an arbitrary number of rough interfaces are obtained in a form, convenient for numerical simulations. In the case of long-range ocean reverberation, several approximations for these expressions are developed, relevant for various environmental scenarios and different types of interfaces, sea-surface, water-sediment interface, buried sediment interfaces, and bottom basement. The results are presented in a simple form and provide a direct relationship of reverberation intensity with three critical characteristics defined at each interface: (1) local spectra of small-scale roughness, (2) local contrast of acoustic parameters, and (3) two-way full-field transmission intensity calculated taking into account only large-scale changes of the environment.

15. SUBJECT TERMS
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